

Standards of Quality and Effectiveness for Programs of Supplementary Authorization in Mathematics

This document contains standards for the development of preparation programs leading to a supplementary authorization in mathematics for teachers who hold a basic teaching credential in another subject area. Such programs may be developed and administered by a school district, county office of education, regionally accredited California college or university, or a consortium of such agencies. This path to obtaining a supplementary authorization in mathematics has been established *in addition* to the existing 20-semester-unit method of obtaining a supplementary authorization. The 20-semester-unit method continues to be available. Agencies interested in preparing a proposed program based on these standards may obtain more information about the proposal submission process by contacting Jim Alford at jalford@ctc.ca.gov.

Standards of Quality and Effectiveness for the Supplementary Authorization in Mathematics

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Introduction to the Standards

This document sets forth program standards for the design, implementation, and operation of programs that recommend the Supplementary Authorization in Mathematics. Each standard is presented in three parts: (a) a succinct statement of the standard itself; (b) a brief rationale explaining why the standard is important; and (c) questions to consider to be used by program designers in developing programs.

The supplementary authorization in mathematics is now defined by course content (through algebra I or integrated course I) rather than grade level (through 9th grade). In the past, supplementary authorizations were based on the supposition that “First-Year Algebra” was the common content for students in the 9th grade. *The Mathematics Content Standards for California Public Schools Kindergarten Through Grade Twelve* and the corresponding *Mathematics Framework for California Public Schools K-12* have set a goal for mathematics instruction that implies that high school geometry is to be the common content of students in the 9th grade. It was never the intent of the supplemental authorization in mathematics to authorize teachers to teach geometry. Geometry courses and those that follow should be taught by single subject credential holders in mathematics. Therefore, the program standards contained in this document are for preparing teachers of mathematics through Algebra I / Integrated Course I, without regard to grade level.

The standards are clustered into two broad categories. Category I addresses the program philosophy and design, subject matter content, and assessment to be covered in the program (Standards 1 through 8). Category II addresses program development and implementation (Standards 9 through 15). Institutions of higher education who have CTC-approved single subject programs in mathematics need only address Category I standards, since the Category II standards were addressed in their single subject program documents.

Standards 2 through 8 ensure that the recipients of the authorization are competent regarding the concepts, skills, and understandings of both mathematics and mathematical pedagogy through Algebra 1 or Integrated Course 1.

The mathematics for grades K-7 described in the *Mathematics Content Standards for California Public Schools Kindergarten Through Grade Twelve* forms the foundation on which standards 2 – 7 are based. The mathematical content of these standards will enable participants to develop an advanced viewpoint of school mathematics through Algebra 1 or Integrated Course 1. Building an advanced viewpoint requires engagement with the reasoning, structures, and interconnections in mathematics that unifies the Academic content standards. This, in turn, enables the participants to examine the overarching concepts of mathematics across the sub-disciplines of mathematics.

Though the mathematics of Standards 2 through 7 are listed by separate subdisciplines, both the specific mathematics detailed and good mathematical pedagogy encourage an integrated view of the connections across mathematics and its applications. In

particular, the mathematical reasoning standard requires participants to demonstrate reasoning processes across all subdisciplines.

In designing a program to meet these standards, it is expected that the content will be presented in ways that model good mathematics teaching. Such teaching takes many forms. It engages the participants in examining, representing, transforming, solving, proving, and communicating mathematics. This can occur when participants learn mathematics in a variety of settings; for example, whole class, collaborative teams, and individually. Participants use the tools of mathematics to develop and enhance their understanding of the content of mathematics and how that content can be taught most effectively. Technology, including calculators and computers, should enhance instruction and active learning of mathematics.

Pedagogy is an essential component of a professional development program in mathematics. Standard 7 details the program requirements in mathematical content pedagogy. In addition to learning mathematics as their students should learn it, participants need to reflect on the instructional decisions that are made during the course of a lesson. They need to analyze why particular methodologies, models, examples, and questions were selected and how these inhibited or fostered the learning of mathematics. In addition, they need to adapt the methods modeled throughout the program to appropriate methodologies that take into account the developmental and learning needs of young adolescents. Participants also need to know how the content of Standards 2 through 7 relate to the mathematics they will teach.

The standards are intended as a guideline for program design and implementation. The sponsoring organization is expected to create a professional development program and evaluation plan based on these standards.

Participants meeting these high standards and receiving supplemental authorizations through programs based on these standards will be better able to provide high quality mathematics instruction to their students.

Category I

Program Philosophy, Design, and Subject Matter Content

Standard 1: Program Philosophy, Purpose, and Design

The program is based on an explicit statement of philosophy that expresses the program purpose and desired outcomes. The design is responsive to the variety of mathematical preparations and needs of the participants. The program curriculum reflects and builds on the *Mathematics Content Standards* and the major themes of the *California Mathematics Framework*. The desired outcome is a teacher who is well prepared to teach the mathematics content through Algebra 1 or Integrated Course 1.

Rationale

An explicit statement of philosophy and program design assists in identifying program goals and needs, direction for program design, course development, and program reviews.

Questions to Consider

- To what extent do the sponsoring organizations, program leadership, and subject matter and education experts collaboratively develop the program philosophy, purpose, and desired outcomes?
- How does the California Mathematics Framework that emphasizes a balanced approach in the teaching and learning of mathematics guide the program philosophy and design?
- How are the expected program outcomes for participants defined clearly so participant assessments and program reviews can be aligned appropriately with program goals?
- In what ways does the organization periodically review and reconsider the program philosophy in light of local program evaluations, ongoing research on the mathematical preparation of teachers, and the changing needs of public schools in California?
- How does the program balance opportunities for participants to gain knowledge in mathematics, reflect on the teaching of mathematics in light of this knowledge, and grow professionally through connections to the mathematics education community?
- How does the mathematical content include and expand upon the ideas in the K-7 Mathematics Content Standards?
- How does the program design assure that the mathematical content is delivered using a variety of instructional strategies?
- In what ways does the program meet other criteria specified by local conditions as appropriate to this standard?

Standard 2: Number Sense

Each program requires participants to demonstrate an understanding of number sense. This should include, but is not limited to, properties and structure of the real number system, appropriate use of estimation and precision, and multiple representations of numbers.

Rationale

Number sense is requisite to an understanding of the algebraic structure of mathematics. It leads to an appreciation of the way different aspects of mathematics are connected and relate to real-world situations. The use of numbers to communicate ideas and information is an essential skill for everyone.

Questions to Consider

In what ways does the program require participants to:

- demonstrate mastery of the arithmetic of the real numbers?
- demonstrate conceptual understanding of the standard algorithms of arithmetic?
- create a variety of representations of a single situation (e.g., geometric, set, algebraic, symbolic, graphical)?
- solve application problems including, but not limited to, reasonableness of answer and estimation, significant number of digits, and scientific notation?
- demonstrate understanding of relative magnitude of both very large and very small quantities?
- understand and apply the structure and properties of real numbers?

These program standards assume that candidates will have a mastery of the California Academic Content Standards in Mathematics, K-7 and Algebra. The supplementary authorization is intended for authorization to teach up to and including algebra I and integrated course I.

Standard 3: Algebra and Functions

Each program requires participants to demonstrate an understanding of the development of functions and algebra up to and including the underpinnings of calculus. This should include the study of algebra as language, algebra as process, algebra as structure, and algebra as modeling. Participants should be able to analyze by direct calculation, and through the use of graphing technology, standard functions and how transformations change the graphs of those functions.

Rationale

Algebra is important as a means of mathematical communication, as generalized arithmetic, and as a tool for solving real-world problems. Teachers need to understand the continuous development of the ideas of algebra from early elementary through high school. The process of building on students' sense of number and moving them toward the uses of variables and functions requires insight into the connections between algebra, number, and other areas of mathematics.

Questions to Consider

How does the program require participants to:

- become fluent in the symbolic language of algebra, including the use and meaning of variables, expressions, statements, and functions?
- engage in algebraic processes such as moving from specifics to generalizations, performing operations and their inverses, solving equations and inequalities, and developing algorithms?
- use a variety of mathematical tools (tables, graphs, equations, functions, matrices, vectors) to organize information and model relationships found in real world situations?
- demonstrate an understanding of families of functions (linear, polynomial, rational, exponential, logarithmic, and periodic) and ways to represent them (numeric, symbolic, graphic, verbal)?
- explore the development of functions and algebra through the underpinnings of calculus, including an informal treatment of limits, rates of change, areas under a curve, and continuity?

These program standards assume that candidates will have a mastery of the California Academic Content Standards in Mathematics, K-7 and Algebra. The supplementary authorization is intended for authorization to teach up to and including algebra I and integrated course I.

Standard 4: Measurement and Geometry

Each program requires participants to demonstrate an understanding of the elements of geometry. This should include analysis of geometric figures using methods and results from transformational, coordinate, and synthetic geometry; study of the process and systems of measurement; properties and relationships of shape, size, and symmetry in two-and three-dimensional space.

Rationale for Standard

Geometry provides a means for visualizing, analyzing, and measuring objects in the physical world, as well as a rich context for the development of mathematical reasoning. Powerful mathematical connections to other mathematical content areas can be made through the use of coordinate and transformational representations.

Questions to Consider

In what ways does the program require participants to:

- precisely describe, classify, and compare types of plane and solid figures according to their attributes?
- use the relationships of congruence and similarity?
- examine proofs of historically important theorems?
- link algebraic and geometric representations using coordinate methods?
- use ratios and proportions to solve problems involving scale factors and similar figures in two- and three-dimensions (perimeter, area, volume)?
- apply transformational techniques of reflection, rotation, translation, and dilation in coordinate and synthetic settings?
- select and use appropriate techniques and tools for measurement?
- determine the level of precision appropriate for particular situations?

These program standards assume that candidates will have a mastery of the California Academic Content Standards in Mathematics, K-7 and Algebra. The supplementary authorization is intended for authorization to teach up to and including algebra I and integrated course I.

Standard 5: Statistics, Data Analysis, and Probability

Each program requires participants to demonstrate an understanding of the processes of gathering and analyzing data and the theoretical constructs that underlie both statistics and probability. This should include emphasis on the following essential components: collection, representation, analysis, and interoperation of data; modeling univariate and bivariate data; and applications of chance and probability.

Rationale

Probability and statistics are fundamental to many disciplines and careers. There is an increasing need for all citizens to organize, analyze, and interpret data in order to make sense of the world around them.

Questions to Consider

To what extent does the program require the participants to:

- demonstrate an understanding of and applications of concepts, principles, and mechanics of data collection such as sampling techniques and measurement of quantitative and qualitative variables?
- calculate, interpret, and apply measures of central tendency, dispersion, and relative standing?
- demonstrate conceptual understanding of the properties of the normal distribution?
- demonstrate an understanding of the effects of various modifications of data sets?
- demonstrate an understanding of and be able to approximate the line of best fit and make predictions from it?
- construct, interpret, and judge the appropriateness of graphical and tabular representations of qualitative and quantitative data sets?
- discuss misleading data displays and abuses of statistics?
- plan and conduct experiments and simulations to determine experimental probabilities?
- develop counting and other techniques useful in determining theoretical probabilities including conditional probability, expected value, and odds?

These program standards assume that candidates will have a mastery of the California Academic Content Standards in Mathematics, K-7 and Algebra. The supplementary authorization is intended for authorization to teach up to and including algebra I and integrated course I.

Standard 6: Mathematical Reasoning

Each program requires participants to demonstrate a variety of reasoning skills in all of the content areas addressed in this document. This should include, but is not limited to, the ability to make conjectures based on an analysis of examples, construct and critique both informal and formal proofs, interpret and use formulas and theorems, use multiple approaches to solve a given problem, and differentiate between inductive and deductive reasoning.

Rationale

Reasoning is fundamental to knowing and doing mathematics. It is essential that an emphasis on reasoning pervade all mathematical activity in order to give individuals access to mathematics as a powerful way of making sense of the world.

Questions to Consider

To what extent does the program require participants to:

- reason in a variety of mathematical content areas, such as algebra, number, geometry and measurement, and probability and statistics?
- use both inductive and deductive reasoning?
- present both informal and formal proofs in both oral and written forms?
- explore and share multiple ways of solving a given problem?
- analyze and discuss the reasoning they use?

These program standards assume that candidates will have a mastery of the California Academic Content Standards in Mathematics, K-7 and Algebra. The supplementary authorization is intended for authorization to teach up to and including algebra I and integrated course I.

Standard 7: Mathematical Content Pedagogy

Each program requires participants to connect mathematical content knowledge to the practice of teaching school mathematics.

Rationale

All mathematics teachers must link content knowledge with effective pedagogy. This linkage gives teachers the versatility to provide all students with access to the concepts and procedures of mathematics. This supports the development of students' mathematical reasoning.

Questions to Consider

To what extent does the program require participants to:

- reflect on the transition from being a learner of mathematics to being a teacher of mathematics?
- examine school instructional materials and analyze the connections between the mathematics of the program and the mathematics they will teach?
- reflect on the ways that their knowledge of the continuum of mathematical content can facilitate their students' conceptual understanding?
- adapt the methods modeled throughout this program to appropriate methodologies that take into account the developmental and learning needs of young adolescents?
- reflect on and discuss the following: choosing appropriate tasks, tools, and materials; employing meaningful strategies; establishing a positive and supportive environment conducive to learning mathematics; facilitating classroom discourse through effective questioning strategies?
- analyze student work as a means to refine instruction and correct student misconception

These program standards assume that candidates will have a mastery of the California Academic Content Standards in Mathematics, K-7 and Algebra. The supplementary authorization is intended for authorization to teach up to and including algebra I and integrated course I.

Standard 8: Assessment of Subject Matter Competence

The program uses multiple measures to assess the subject matter competence of each participant formatively and summatively in relation to the content of standards 2 to 7 and the K-7 Academic Content Standards. The scope and content of each participant's assessment is consistent with the studies the participant has completed in the program. The assessment information is also used to inform the scope, focus and content of the program.

Rationale

An organization that offers a program leading to a supplementary authorization in mathematics has a responsibility to verify the mathematical competence of its participants. To address the needs of all participants, it is essential that the assessment use multiple measures, have formative and summative components, and be as comprehensive as standards 2 to 7.

Questions to Consider

- How does the program provide avenues for addressing deficiencies in mathematical background or experience of the participants?
- To what extent does the assessment include a variety of approaches, such as participant presentations, projects, portfolios, observations and interviews, and oral and written examinations?
- How does the assessment encompass the content of standards 2 to 7 and the K-7 Academic Content Standards?
- How is the assessment aligned with the participant's actual studies in the program?
- How is the assessment made valid, reliable, equitable, and fair, and how does it include provisions for appeals?
- In what ways are the assessment scope, process and criteria clearly delineated and available to participants?
- How does the organization make and retain records regarding each participant's performance in the assessment?
- How does the assessment address other program qualities, as specified by local conditions, which are appropriate to this standard?

These program standards assume that candidates will have a mastery of the California Academic Content Standards in Mathematics, K-7 and Algebra. The supplementary authorization is intended for authorization to teach up to and including algebra I and integrated course I.

Category II

Program Development and Implementation

Standard 9: Program Coordination

The program is sponsored by one or more organizations that demonstrate a commitment to the mathematical and professional growth of the participants. The program has strong leadership and an administrative structure organized, governed and coordinated with the active involvement of mathematics and mathematics education experts. Program leaders have appropriate authority over the details of program design and implementation.

Rationale

The commitment of sponsoring organizations (school districts, county offices of education, professional organizations, institutions of higher education, etc.) is indispensable for the effectiveness and durability of the program. The accomplishments of participants in supplemental authorization programs depend in part on the effective coordination of the program by responsible members of these organizations. For participants to become competent in mathematics and the teaching of mathematics, all aspects of their subject matter preparation must be planned thoughtfully and implemented conscientiously.

Questions to Consider

- What is the mechanism for effective communication and coordination among the sponsoring organizations, the program leadership, and the subject matter specialists responsible for the program?
- In what ways are the responsibilities of each sponsoring organization for program oversight and implementation clearly specified, and a primary contact person designated for each?
- How are linkages made to provide opportunities for collaboration among a variety of organizations such as school districts, county offices of education, professional development centers, mathematics project sites, college or university mathematics and education departments, and professional organizations?
- Is a program director appointed to assume responsibility for overall direction of the program; are the roles, responsibilities and time commitment of the program director clearly defined and appropriate to the scope of the program?
- How does the program meet other criteria specified by local conditions as appropriate to this standard?

Standard 10: Program Resources

The sponsoring agencies consistently allocate sufficient personnel time, including support personnel, fiscal resources and space to administer and conduct the program and fulfill standards 2 through 7. Sufficient resources are allocated for program curriculum, instruction, and assessment. Sufficient resources are also allocated for faculty development.

Rationale

A program's resources affect its quality and effectiveness. Lack of resources impedes participants from achieving high standards of quality or competence.

Questions to Consider

- How are adequate personnel resources equitably provided to maintain an effective program?
- How do the program's personnel and participants have access to appropriate buildings, classrooms, offices, professional services, instructional materials (including print material and technology-based instructional tools), and adequate clerical support?
- How does the program meet other criteria specified by local conditions as appropriate to this standard?

Standard 11: Faculty

The faculty providing instruction in this program are qualified by experience and formal education to deliver the necessary mathematical content, work with adult learners, model a variety of pedagogical approaches, engage participants in reflective practices regarding the teaching and learning of mathematics, and demonstrate a variety of assessment practices.

Rationale

Content competency is a necessary component for program faculty. It also is essential that faculty involved in providing instruction in mathematics understand the variety of ways that mathematics is learned and use a variety of strategies that will help make mathematics accessible to all learners. Consequently, the instruction in the program will combine significant mathematical content with exemplary classroom practices and promote the implementation of these practices within participants' classrooms.

Questions to Consider

- Do the instructors include some combination of fully certified mathematics teachers, professors of mathematics education, professors of mathematics, and other qualified mathematics educators?
- To what extent are the instructors knowledgeable about the *California Mathematics Framework* and the *Mathematics Content Standards for California Public Schools Kindergarten Through Grade Twelve*?
- Do the instructors include people with experience teaching middle or high school students in mathematics classrooms?
- Do the instructors include people with experience delivering professional development for mathematics teachers?
- Do the instructors include people with experience teaching college or university mathematics or mathematics education?
- To what extent do the instructors have experience using a variety of instructional and assessment strategies?

Standard 12: Admissions, Advice, and Assistance

The program has a fair and equitable selection process based upon well-defined criteria. Participants will be admitted whose professional experiences suggest the highest potential for success as a mathematics teacher. The provider makes available to each teacher in the program all requirements, standards, and procedures that affect their progress toward attaining the supplemental authorization in mathematics. Qualified members of the program staff are available to advise teachers about their academic and professional development as the need arises. The program assists candidates who need special assistance and recommends only those candidates who satisfy the requirements of the program.

Rationale

The program needs a clear application process to examine the eligibility of potential participants and to assist in the selection process. The program has the obligation to provide the teachers with accurate and relevant information and to give teachers appropriate feedback on their progress.

Questions to Consider

- How are the selection criteria and procedures clearly described and available to prospective participants?
- In what ways does the selection process reflect a commitment to achieve a balanced representation of the population?
- How does the program insure that all current and prospective participants receive relevant information about the program and the availability of assistance?
- How does the program encourage participation of members of diverse populations?

Standard 13: Equity

The program promotes educational equity through its instructional, advisement and curricular practices. Each participant in the program acquires knowledge and appreciation of the perspectives and contributions of diverse cultural, ethnic and gender groups in the area of mathematics. Participants experience classroom practices and use instructional materials that promote educational equity among diverse learners.

Rationale

Students who attend California schools are increasingly diverse. They live in a society that has benefited from the perspectives and contributions of men, women, and many cultural, ethnic and gender groups. Teachers must understand and appreciate the cultural perspectives and intellectual contributions of these groups. They must also be aware of barriers to academic participation and success, and must encourage equitable practices of education during their preparation.

Questions to Consider

- How are both men and women, members of culturally and ethnically diverse groups, and individuals with exceptional needs encouraged to apply to the program as faculty members?
- How are both men and women, members of culturally and ethnically diverse groups, and individuals with exceptional needs encouraged to enter and complete the program?
- To what extent do participants learn about the contributions and perspectives of diverse cultural, ethnic and gender groups related to significant mathematical subjects?
- To what extent do participants examine practices in mathematics education that restrict participation of many groups in mathematics or mathematics related study and careers?
- How does coursework in the program foster understanding, respect and appreciation of human differences in the learning of mathematics?
- Does the program have other qualities related to this standard that are brought to the reviewers' attention by the institution?

Standard 14: Program Evaluation

The agencies that sponsor the program operate a comprehensive, ongoing system of program evaluation that leads to continuous improvement of the program.

Rationale

To achieve high quality and effectiveness, a program should be evaluated regularly and comprehensively and refinements made based on the evaluation results. The evaluation should be based on the Program Standards as well as local goals. The evaluation should support continued development of the program, and should recognize that both formal and informal evaluations serve useful purposes.

Questions to Consider

- In what ways the program systematically and objectively evaluated on the basis of criteria that are related to the design, rationale, goals and objectives of the program?
- How is evaluative input collected from a variety of stakeholders, including: program participants, community members, graduates, local mathematics teachers, faculty, administrators?
- In what ways are data collected on program participants, graduates and faculty?
- How are adjustments and improvements in all components of the program based on the results of program evaluation, the implications of new knowledge about teaching and learning, and the identified strengths and needs of the participants?